INTERNATIONAL DAY OF MATHEMATICS
MARCH 14

Paper Activities

In this document, you will find several activities making use of paper:

1. Coloring Maps
2. Fish Octahedron
3. Tessellations
4. Bridges of Königsberg
5. Fold-and-Cut

Participants:

Ages 10 - 12 and up (depending on the activity).
No previous mathematical knowledge is required.

Preparations:

Printed templates (one for each participant), coloring pencils.
Some activities require scissors and glue or tape.

Activity 1. Coloring Maps:

The rules for coloring different regions are:
Two sections that share a common edge cannot be colored with the same color.
Ask participants to color each figure with as few colors as possible.
Options: Have local maps printed out at hand. Play with this interactive app for coloring maps: https://mathigon.org/course/graph-theory/map-colouring

Activity 2. Fish Octahedron:

The template gives the net (in two parts) for an octahedron decorated with tessellated fish. Print two copies, cut along solid lines and fold on dotted lines. Color if desired and glue or tape the two halves together.
Introduce the concept of a polyhedra (vertices, edges, faces) and find examples around you.
Ask questions:
How many faces, edges and vertices does the paper sculpture have, and how many fish? How many mouths meet at each vertex? How many tails meet and where?

Where do the fins meet? (Two mouths will meet at each vertex, three tails at the center of four faces, and three fins at the center of the other four faces.) What other closed sculptures can you build from (equilateral) triangles?
Count the number of faces (F), edges (E) and vertices (V) of several different convex polyhedra. Put them in a table visible to all and try to find Euler's Polyhedron formula: V - E + F = 2. Give hints when needed (i.e. only use addition and subtraction).

Activity 3. Tessellations:

Have participants color the two images on the template. Talk about mirror-symmetries. Ask, how the pattern would go on after the edges. Look for symmetries in things that surround you. Draw your own pattern, create T-shirts with your design…

Activity 4. Bridges of Königsberg:

A river divides a town into separate areas connected by bridges. Is it possible to walk around the city crossing all of the bridges exactly once (and not more than once)? You can start and finish anywhere, not necessarily in the same place.
Try to find a valid route by drawing on the maps provided.

Activity: 5. Fold-and-Cut:

Take a piece of paper, fold it flat several times and make one single completely straight cut. Unfold the pieces.
Now thinkt: Which shapes are possible to produce with a cut like that?
Try to fold and cut some of the examples given in the template.

Create and Share!
Share the participant's drawings and additional templates you created using the hashtag **#idm314paper** and **#idm314**.

Mathematical background and resources:

Coloring Maps

The Four-Color-Theorem is one of the most famous theorems of mathematics. It states that any pattern or map can always be colored with four colors (or less).
It is important because it was first stated in 1852, but not proved until 1976. For over one hundred and twenty years some of the best mathematical brains in the world were unsuccessful in proving one of the simplest theorems in mathematics. There were many false proofs, and a whole new branch of mathematics - known as Graph Theory - was developed to try to solve the theorem. But nobody could prove the theorem until Appel and Haken proved the theorem in 1976 with the aid of a computer. Some people think that, although their proof was correct, it was cheating to use a computer. What do you think?

Fish Octahedron

A polyhedron is a closed shape in 3D space with flat faces, straight edges and sharp corners (vertices). Examples are the cube or a pyramide. The Octahedron is another example (imagine two square pyramids glued together at the square face). You can imagine it living inside a cube (its corners touch the center of the faces of the cube. You can equally well imagine a cube living inside an octahedron. They are called dual. Try finding the dual of the pyramid.
A polyhedron is called convex, if you stay inside the object when travelling from any point of the surface to any other point of the surface in a straight line. The cube, pyramide and octahedron are examples for convex polyhedra.
Polyhedra can look very different from each other and may have many or few faces, edges and vertices. However, there is one property which is the same (invariant) for all convex polyhedra: Take a convex polyhedron and count its faces (F), edges (E) and vertices/corners (V). If you compute V - E + F, you will always get the same number namely 2, no matter which polyhedron you chose. This number is called the Euler Characteristic and was first stated by the famous mathematician Leohard Euler in 1758.
Note: The Euler characteristic can be different from 2 for nonconvex polyhedra, if they happen to have one or more holes. For convex polyhedra, holes are not possible.

Other Options:

- Play the game MatchTheNet: https://www.matchthenet.de/
- Build other polyhedra using these nets: https://imaginary.org/sites/default/files/matchthenet-polyhedra-nets.pdf
- Have a look at these images of polyhedra and create corresponding nets: https://imaginary.org/sites/default/files/matchthenet-polyhedra-images.pdf

- Check out the Mathigon-course on polyhedra:
  https://mathigon.org/course/polyhedra/polygons
- Adopt a polyhedron:
  https://www.polytopia.eu/en/

Tessellations

A tessellation or tiling is a pattern on a flat surface (ideally an infinitely extended plane) made up of (geometric) shapes called tiles with no gaps or overlaps. If the pattern repeats, it is called a periodic tiling. Otherwise it is non-periodic.
Historically, tessellations were used in Ancient Rome and in Islamic art.
The patterns formed by periodic tilings can be categorized into 17 wallpaper groups.

Other Options:

- Draw your own wallpaper online using iOrnament:
  https://imaginary.github.io/cindyjs-apps/iornament/index.html
- Check out the Mathigon-course on symmetry and transformations:
  https://mathigon.org/course/transformations/introduction

Bridges of Königsberg

One of the first mathematicians to think about graphs and networks was Leonard Euler. He was intrigued by an old problem regarding the town of Königsberg near the Baltic Sea (now Kaliningrad, Russia). In the case of Königsberg it seems to be impossible to find a valid route through all bridges crossed exactly once, but some of the other cities do work.
The problem is closely connected to drawing a figure (graph) without lifting the pen and without tracing the same line more than once. Each of the given city maps can be converted into graphs with edges and vertices: Every island or region of land is represented by a vertex and every bridge connecting two regions is represented by a corresponding edge. Only the connections are relevant, the graph can otherwise be distorted in any way, lines do not have to be straight.

Other options:

Have the kids come up with a few different graphs and then try to work out which ones can be drawn with a single, continuous stroke. It helps to find out the degree of each vertex of a graph (number of edges, which meet at that vertex). Through discussion/brainstorming the group can come up with rules which need to be satisfied for a graph in order that it can be traced as described above. For instance: The graph needs to be connected. Only two vertices may have odd degree. If there are vertices of odd degree, the path needs to start at one of them and end at the other.

A path that visits every edge of a graph exactly once (a vertex may be visited several times) is called “Eulerian Path”. If the path starts and ends at the same vertex, it is called an “Eulerian Cycle". Euler’s Theorem states: “A connected graph has an Euler cycle if and only if every vertex has even degree.” It was proven in 1873 by Carl Hierholzer.
Try the online version here: https://mathigon.org/course/graph-theory/bridges.

Fold-and-Cut

The theorem is that every pattern (plane graph) of straight-line cuts can be made by folding and one complete straight cut. Thus it is possible to make single polygons (possibly nonconvex), multiple disjoint polygons, nested polygons, adjoining polygons, and even floating line segments.
Find more information here: http://erikdemaine.org/foldcut/

Credits and License:

The Coloring Maps activity is based on an activity by Rod Pierce. The images in the template are used with his permission. The original activity can be found at the Website Math is Fun www.mathsisfun.com.

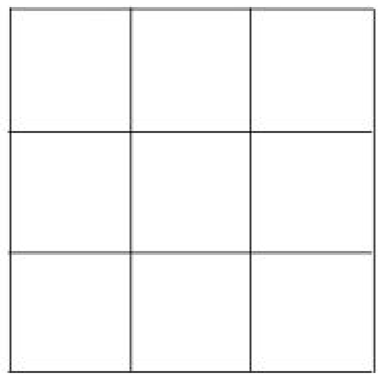
The Fish Octahedron and Tessellations templates were created by Robert Fathauer and are used with his permission.

The Bridges of Königsberg activity is based on a course from Mathigon www.mathigon.org. The images of the maps are used with permission of Philipp Legner.

The Fold-and-Cut activity is based on the description by Eric Demaine given here: http://erikdemaine.org/foldcut/ The templates for this activity were created by Christiane Rousseau.

Template for Coloring Maps (page 1 of 3)

Try different colorings for the same figure or color just one of them:



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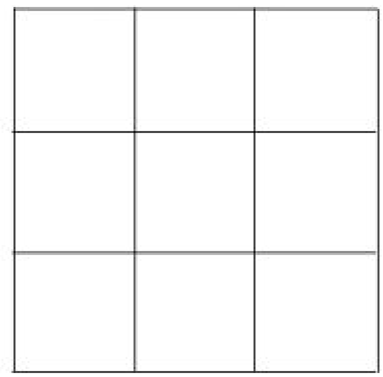

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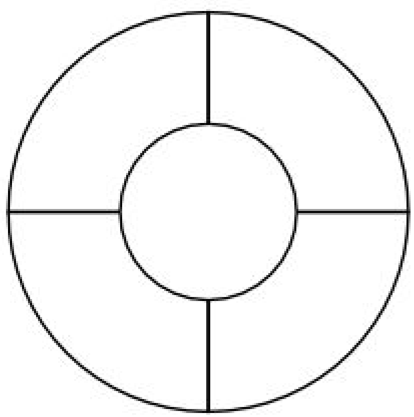

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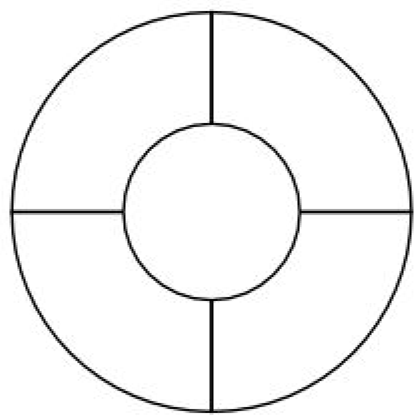

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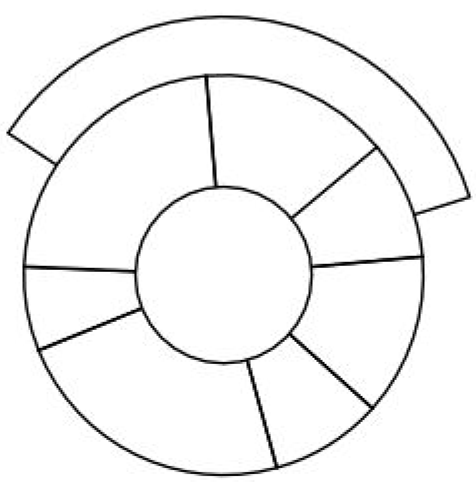

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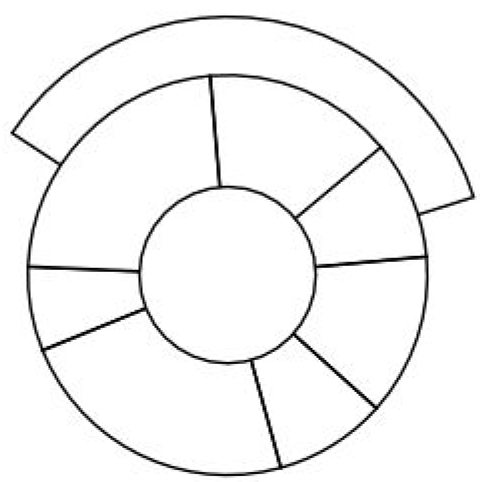

# of colors: ____

Template for Coloring Maps (page 2 of 3)

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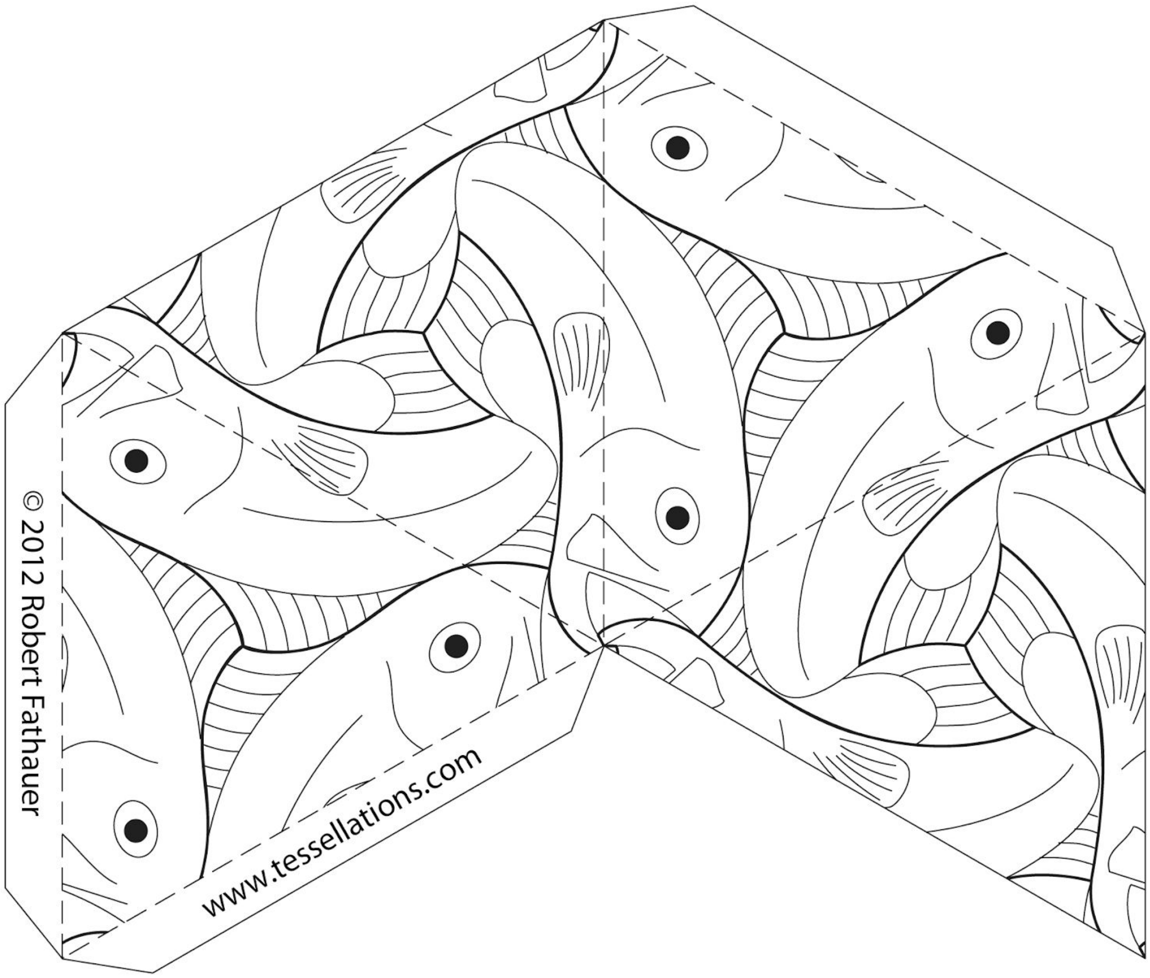
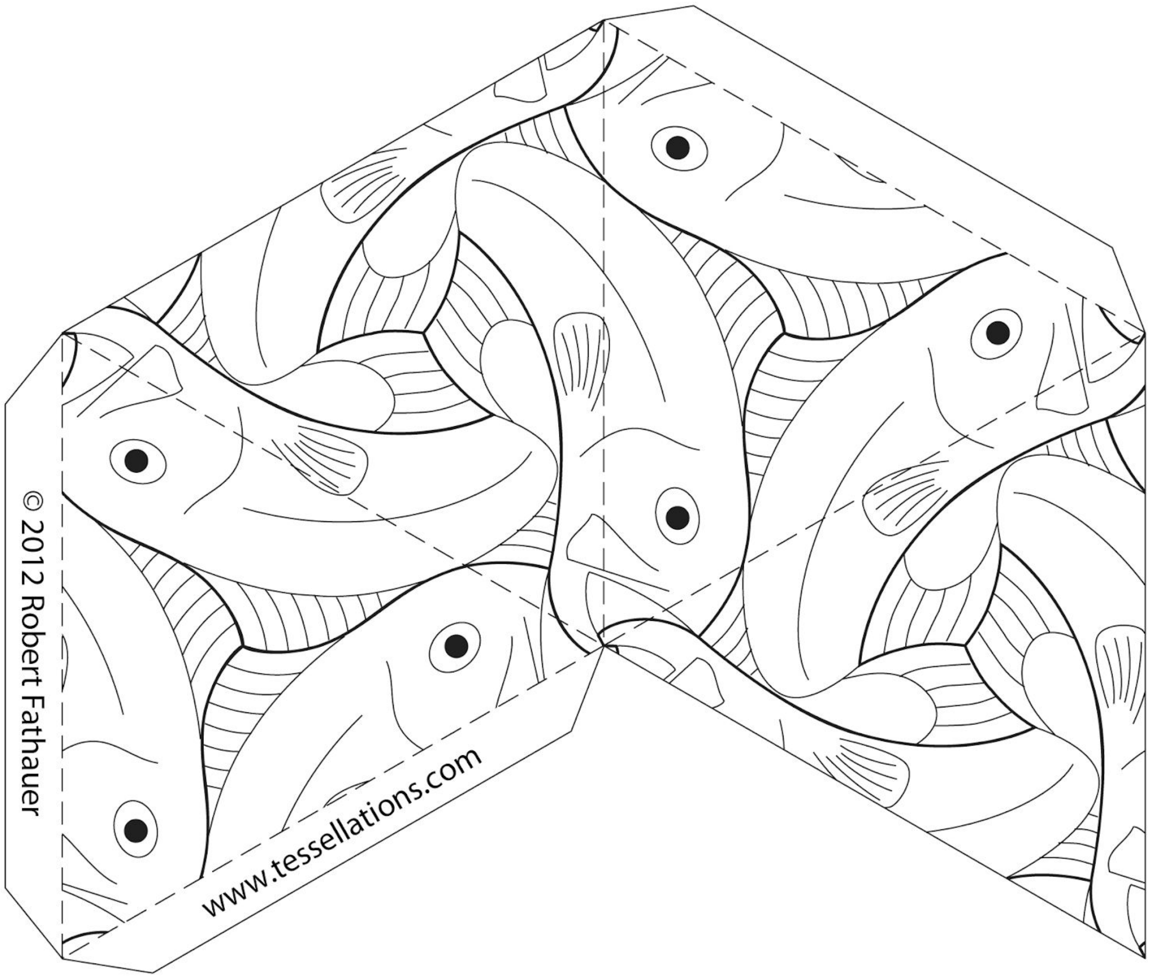
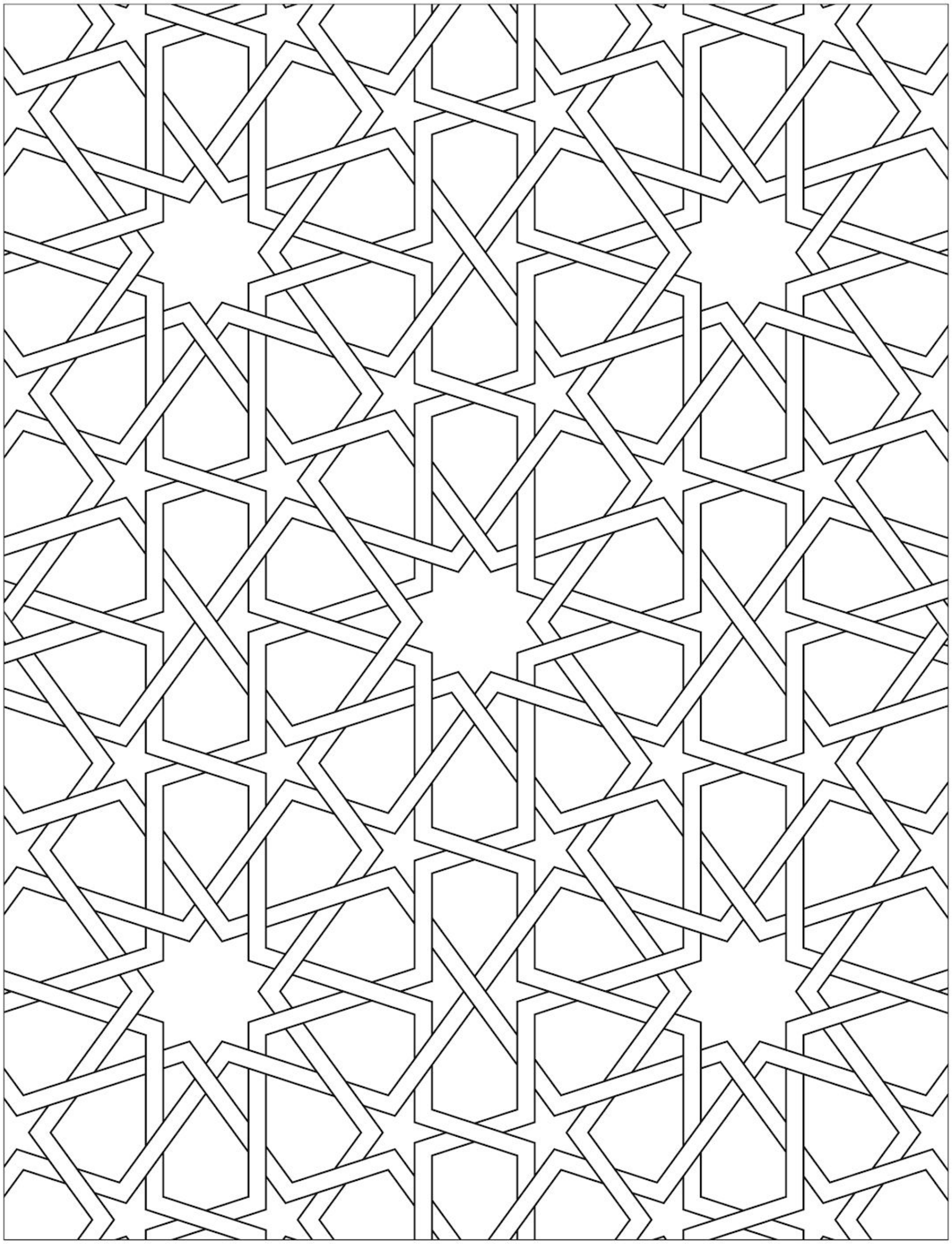
Template for Fish Octahedron (page 1 of 2)



Template for Fish Octahedron (page 2 of 2)

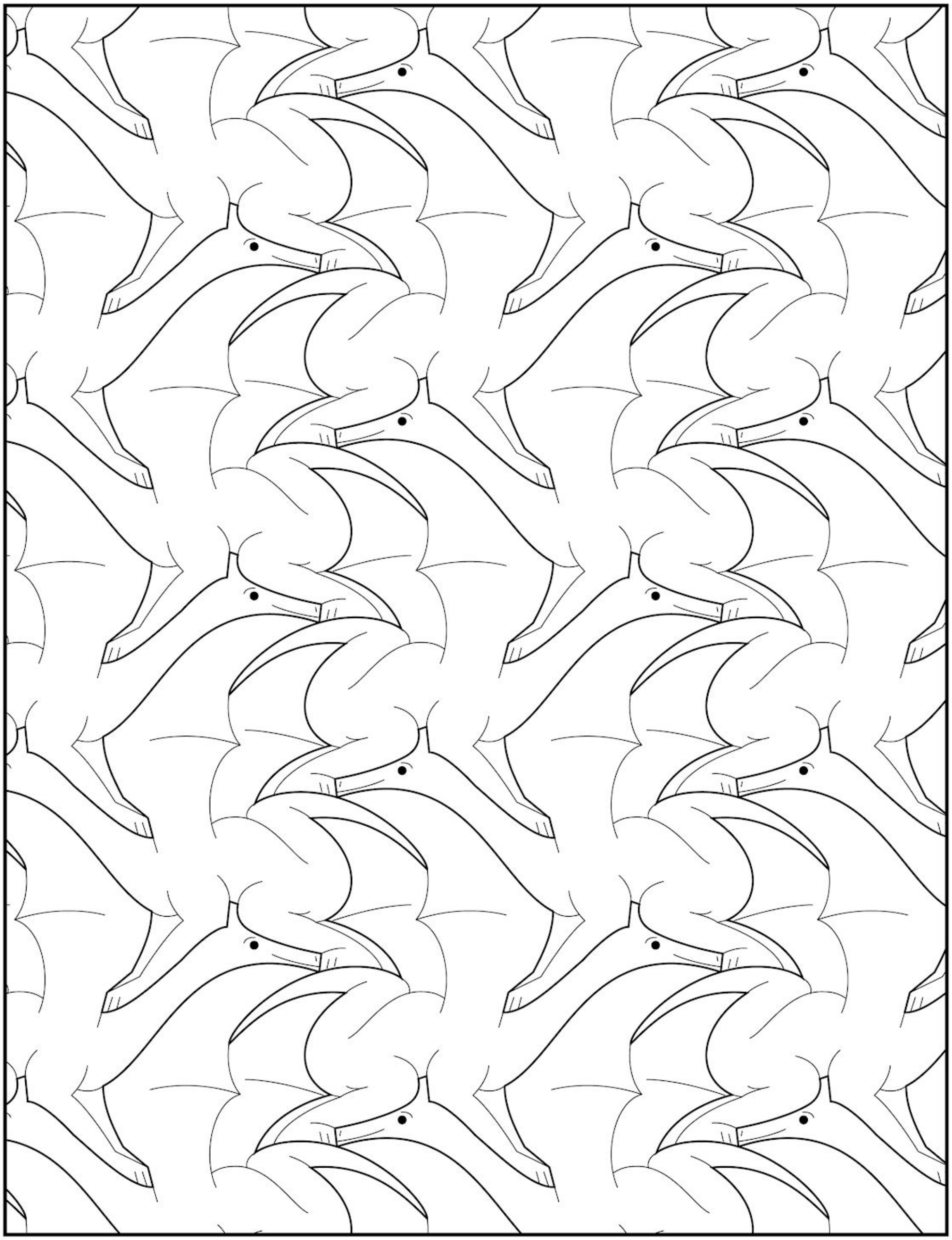


Template for Tessellations (page 1 of 2)



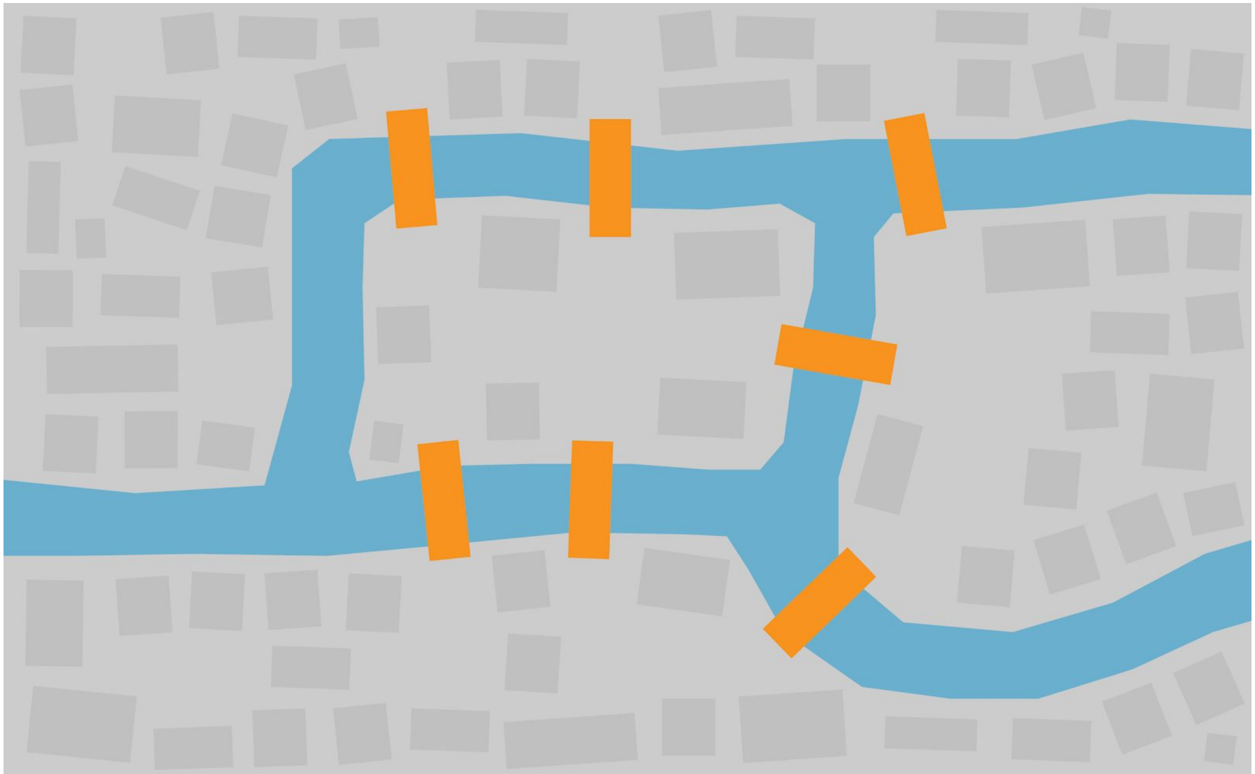
www.tessellations.com

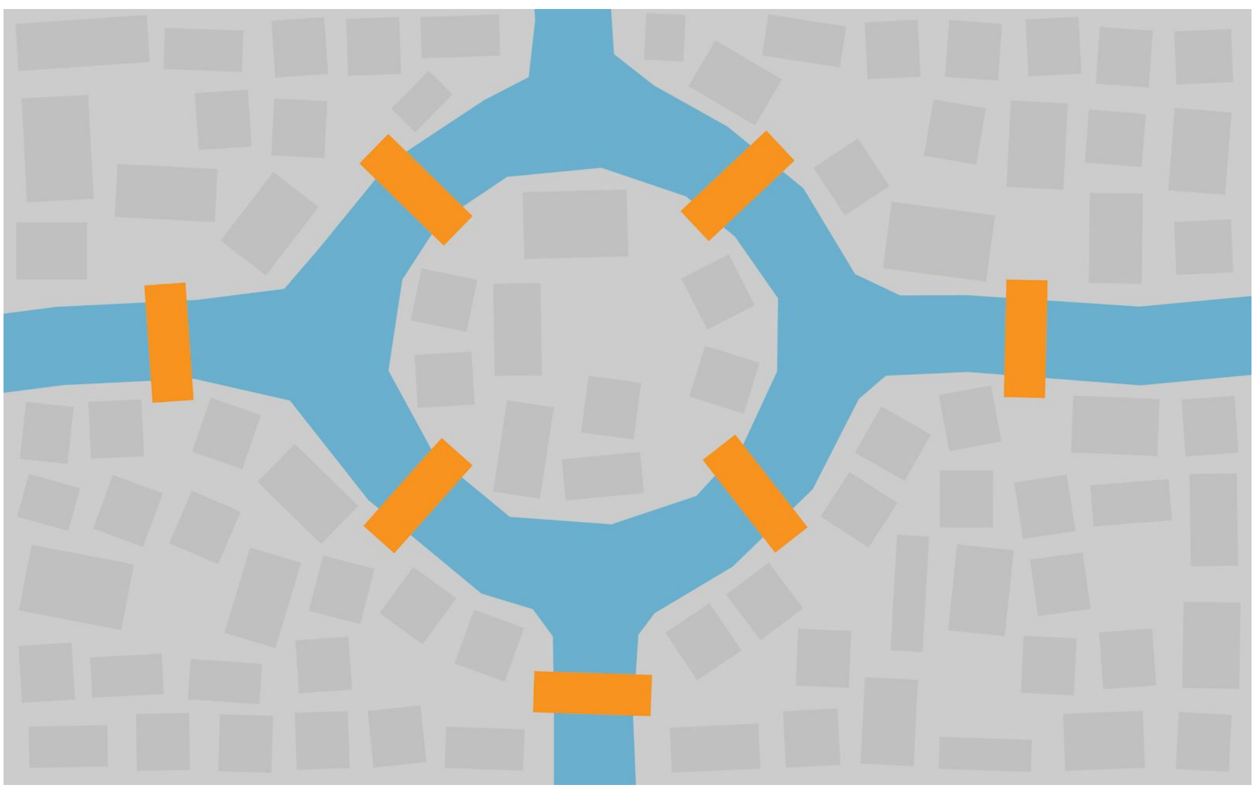
Template for Tessellations (page 2 of 2)



"Dragons"

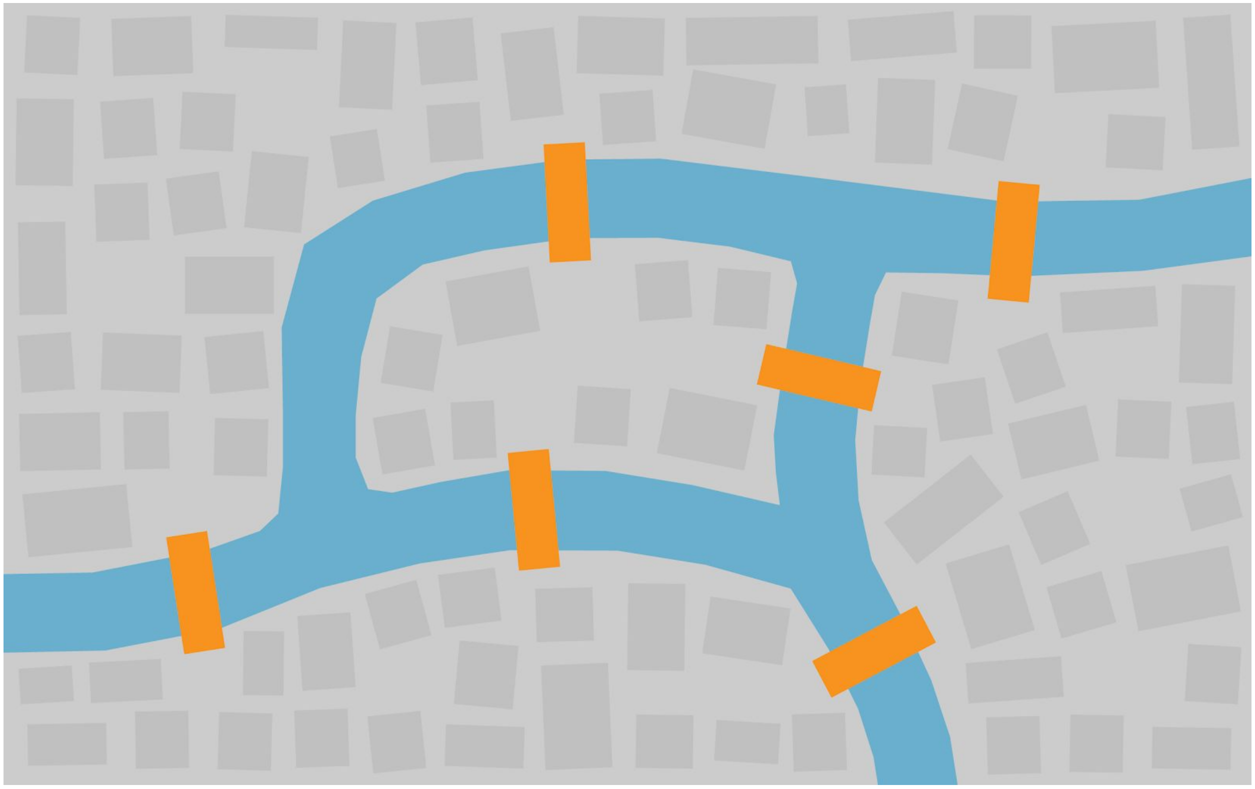
Template for the Bridges of Königsberg (page 1 of 2)

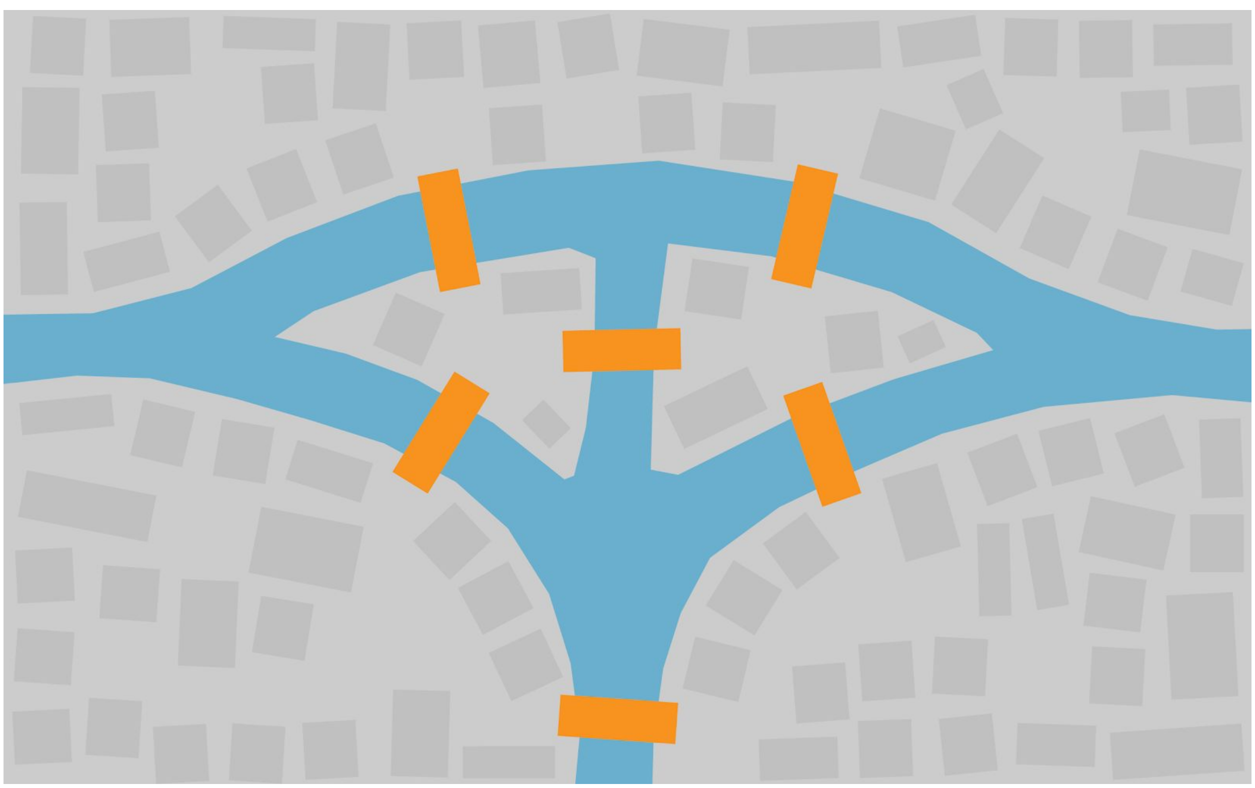




www.mathigon.org

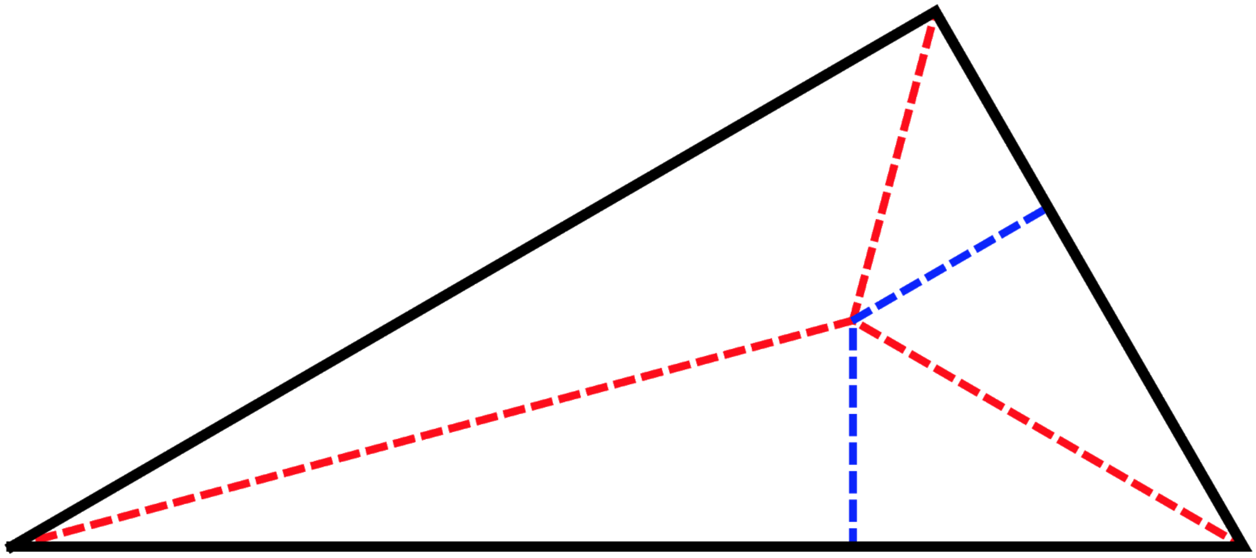
Template for the Bridges of Königsberg (page 2 of 2)

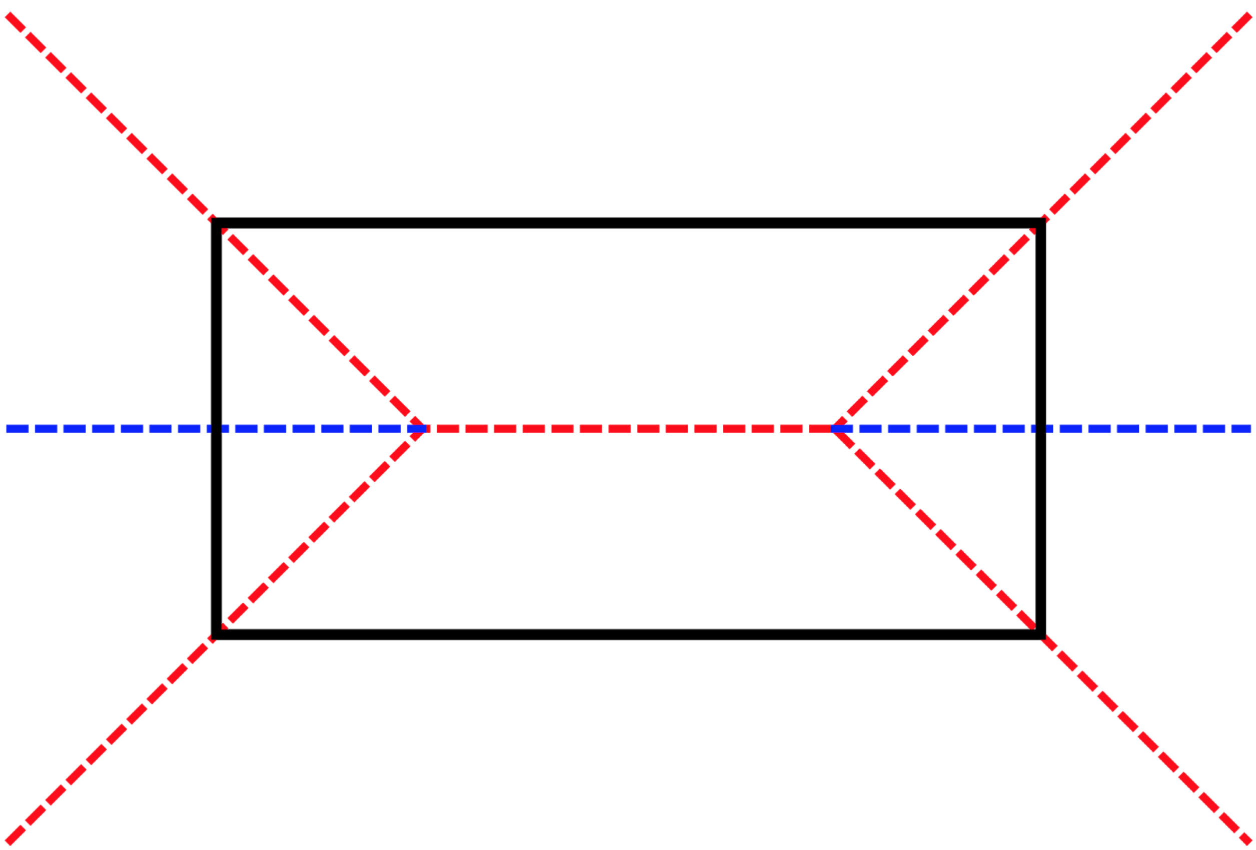




www.mathigon.org

Template for Fold-and-Cut





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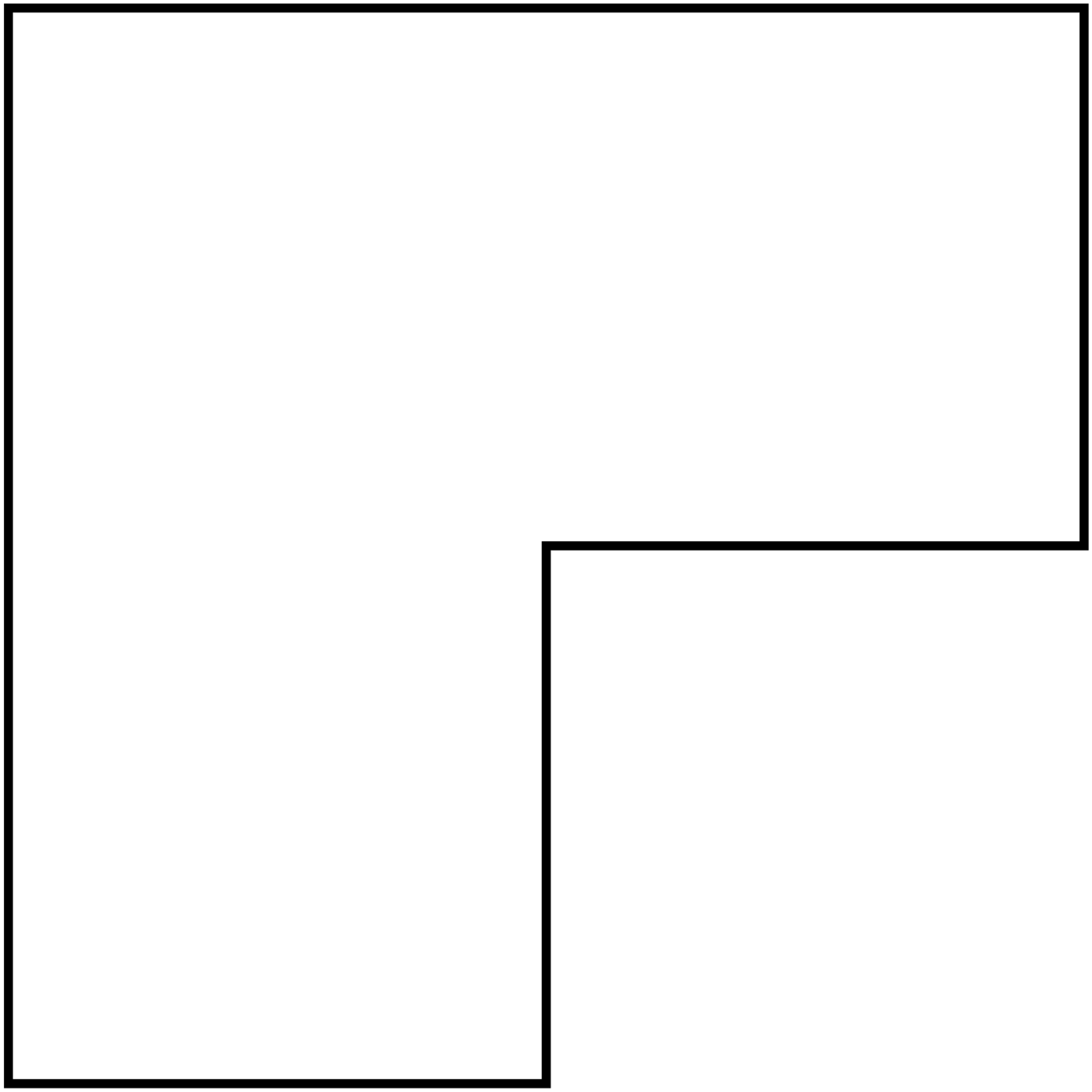

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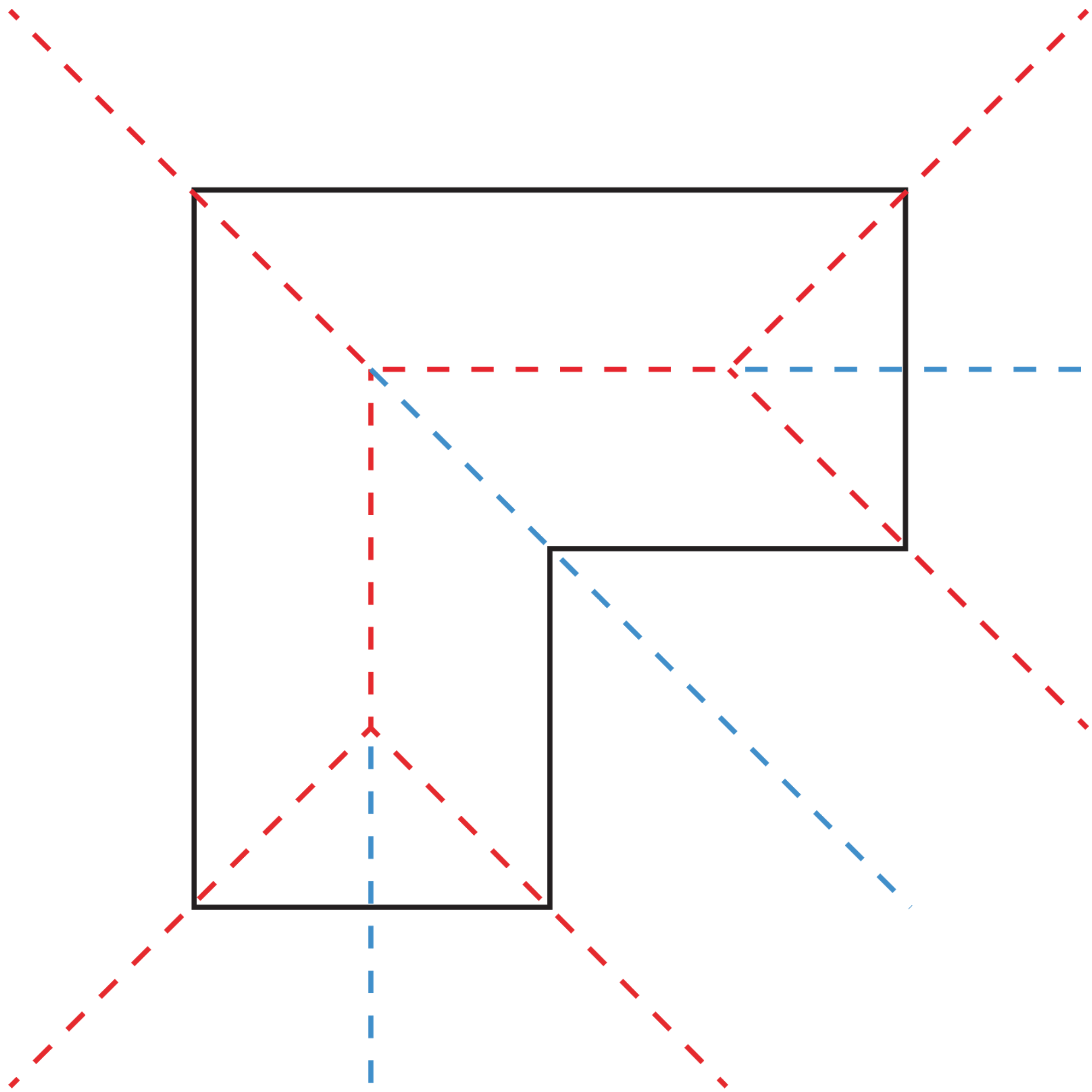

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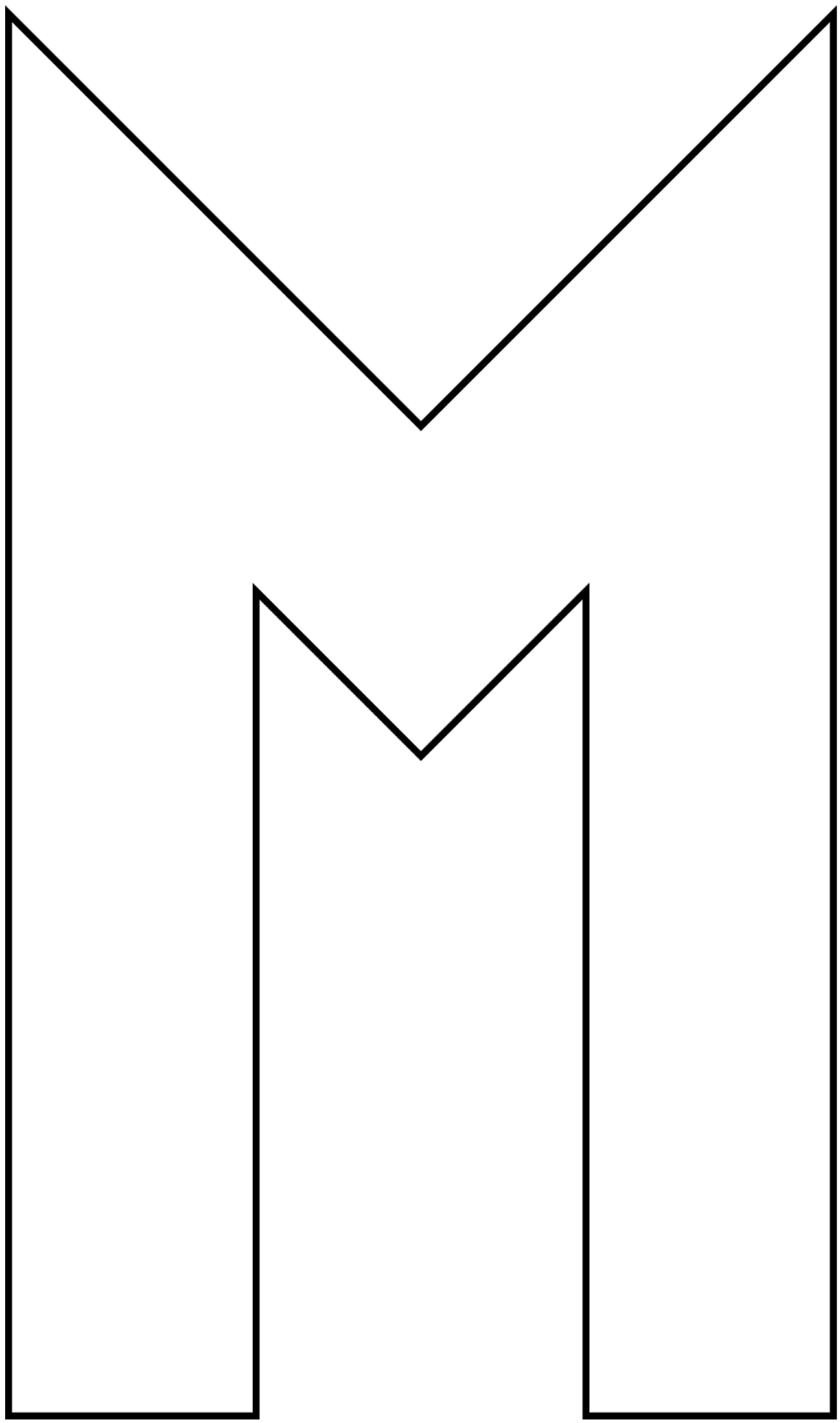

Template for Fold-and-Cut